Influence of uncertainties on the B(H) curves on the flux linkage of a turboalternator

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ABSTRACT

In this paper, we analyze the influence of the uncertainties on the behavior constitutive laws of ferromagnetic materials on the behavior of a turboalternator. A simple stochastic model of anhysteretic non-linear B(H) curve is proposed for the ferromagnetic yokes of the stator and the rotor. The B(H) curve is defined by five random parameters. We quantify the influence of the variability of these five parameters on the flux linkage of one phase of the stator winding depending on the excitation current I. The influence of each parameter is analyzed via the Sobol indices. With this analysis, we can determine the most influential parameters for each state of magnetization (according to the level of I) and investigate where the characterization process of the B(H) curve should focus to improve the accuracy of the computed flux linkage.

KEYWORDS: Uncertainties quantification, Non-linear behavior laws, Stochastic approach, Global sensitivity analysis, Sobol coefficients, Polynomial chaos expansion.

1. INTRODUCTION

Numerical model can be used to predict the behavior of an electrical machine. Basically, the numerical model requires as input data the geometry of the device and the behavior laws of the materials. Uncertainties on the input data can appear due to several factors like the imperfection of the manufacturing process, the ageing of the material, the impacts of the environment (variation of temperature, humidity)... Thus, the output data of the model are also uncertain. Global sensibility analysis allows determining the influence of the variability of each input data on the variability of one or more observed output data. Then, the most influential and the less influential input parameters can be distinguished. Thus, the sensitivity analysis can be useful to several purposes like the reduction of the variability of the output data by reducing the variability of the most influential input data or the reduction of the computational cost by reducing the number of random inputs (the less influential input data can be considered deterministic and equal to a fixed value)…

The probabilistic approach [1] that consists in modeling the uncertain inputs by random variables (or random fields) is one of the most popular among the methods [1-3] for uncertainty quantification. The outputs of the model are then also random variables or fields. Characterization of the random outputs can be obtained using sampling technique as Monte Carlo Simulation Methods. Random outputs can be also approximated using well fitted space as Polynomial Chaos Expansion [6-12]. In the probabilistic approach, the global sensitivity of an observed output can be quantified by an Analysis Of Variance (ANOVA) like the approach proposed by Sobol [4]. In the Monte Carlo simulation method (MSCM), the Sobol coefficients are calculated from a number of realizations of the observed output data [5]. In the chaos polynomial development method, the Sobol coefficients can be deduced directly from the coefficients in chaos polynomial expansion [13].

In electromagnetism, few applications have been already processed to demonstrate the new possibilities provided by such approach especially for global sensitivity analysis. In [14], the influence of the uncertainties of the measured points used to characterize a B(H) curve on the magnetic field distribution created by accelerator magnets is studied. In this paper, based on a similar approach, we aim at analyzing the influence of uncertainties of the non-linear B(H) curve used to model ferromagnetic material on the uncertainty of the predicted behavior of a turboalternator.

The B(H) curve is defined by five parameters (B_1, H_1, B_2, H_2, α) which are the coordinates of two points P1 and P2 and the asymptotic slope of the curve for high values of H. We quantify the influence of the variability of these five parameters on the flux linkage through one of the phases of the stator in function of the excitation current I in the rotor. The influence of each parameter is analyzed using the Sobol coefficients. For each state of magnetization (according to the level of I), we determine the most influential parameters among (B_1, H_1, B_2, H_2, α).

2. STOCHASTIC MAGNETOSTATIC PROBLEM WITH UNCERTAINTIES ON THE BEHAVIOR LAW

We consider a stochastic magnetostatic problem defined on a domain $\mathbb{D}$:

$$\begin{cases} div\,\mathbf{B}(x,\theta) = 0 \\ \boldsymbol{curl}\,\mathbf{H}(x,\theta) = 0 \\ \mathbf{B}(x,\theta) = g(\mathbf{H}(x,\theta),x,\theta) \end{cases} \qquad (1)$$

where $\mathbf{B}(x,\theta)$ and $\mathbf{H}(x,\theta)$ are respectively the magnetic flux density and the magnetic field, θ is an elementary event referring to the randomness and x is the spatial coordinates. The function g represents the material behavior law. The problem (1) is supplemented by some boundary conditions. We assume that the domain $\mathbb{D}$ is deterministic and is composed by several sub-domains and in each sub-domain $\mathbb{D}_i$, the random curve $\mathbf{B} = g_i(\mathbf{H},\theta)$ is independent of the position x. Random geometries can also be considered [15, 16, 17, 18] but they are out the scope of this paper. If the behavior law is assumed linear and random, the curve $\mathbf{B} = g_i(\mathbf{H},\theta)$ is linear with a random slope which is the random permeability $\mu_i(\theta)$. In the general case, the random curve $\mathbf{B} = g_i(\mathbf{H},\theta)$ is a random field. To solve numerically the problem (1), this curve should be represented (or at least approximated by a KL expansion like in [20]) by a function of a finite number of random parameters $\boldsymbol{\xi}(\theta) = (\xi_1(\theta), \xi_2(\theta),..., \xi_M(\theta))$, that is to say $\mathbf{B} = g_i(\mathbf{H}, \xi_1(\theta), \xi_2(\theta),..., \xi_M(\theta))$. The M random variables $\xi_j(\theta)$ have a known probability density function (normal, uniform, etc.) that will be assumed to be independent. We denote $\Theta_j \subset \mathbb{R}_j$ the set of value of $\xi_j(\theta)$, and f_j its probability density function (pdf). We denote also

$$\boldsymbol{\Theta} = \Theta_1 \times \Theta_1 \times \cdots \Theta_M \qquad (2)$$

the set of value of the random vector $\boldsymbol{\xi}(\theta)$ and

$$\boldsymbol{f} = \prod_{j=1}^{M} f_j \qquad (3)$$

its pdf . We then obtain a model with input random variables modeled by a random vector $\boldsymbol{\xi}(\theta) = (\xi_1(\theta), \xi_2(\theta),..., \xi_M(\theta))$ with known pdf, then the fields **B** and **H** are functions of both the position x and the random vector $\boldsymbol{\xi}(\theta)$. In the following, to simplify the notations, the dependency of the random vector $\boldsymbol{\xi}$ on θ will be removed.

We consider now a quantity of interest $Q = Q(\xi)$ (magnetic energy, mechanic torque, magnetic flux...). To characterize $Q(\xi)$, sampling technique like the Monte Carlo Simulation Method can be used. Another possibility consists in approximating $Q(\xi)$ in a finite dimension space of functions of ξ. Several methods were proposed in the literature to approach this expression [7-11] and [19]. Among these methods, polynomial chaos [12] is widely used. The approximation of $Q(\xi)$ is obtained in the following form:

$$Q(\boldsymbol{\xi}) \approx Q^p(\boldsymbol{\xi}) = \sum_{\boldsymbol{u}\in K_p} c_{\boldsymbol{u}} \Psi_{\boldsymbol{u}}(\boldsymbol{\xi}) \qquad (4)$$

where $\Psi_{\boldsymbol{u}}(\boldsymbol{\xi})$ are multivariate orthogonal polynomials [12], $c_{\boldsymbol{u}}$ are the coefficients to determine and the set K_p of M-tuples is defined by:

$$K_p = \left\{ (u_1, u_2, ..., u_M) \in \mathbb{N}^M \mid \sum_{i=1}^{M} u_i \leq p \right\} \qquad (5)$$

The multivariate polynomials $\Psi_{\boldsymbol{u}}(\boldsymbol{\xi})$ are obtained from monovariate orthogonal polynomials. If we denote $(\psi_{uj}(y))_{uj\in \mathrm{N}}$ the set of orthogonal polynomials according to the weight function f_j (f_j is the pdf of ξ_j), then the multivariate polynomial is given by:

$$\Psi_{(u_1, u_2, \cdots, u_M)}(\xi_1, \xi_2, \cdots, \xi_M) = \prod_{i=1}^{M} \psi_{u_i}(\xi_i) \qquad (6)$$

Example. *Let ξ_1, ξ_2 are standard normal random variables, the monovariate polynomials $(\psi_{uj}(y))_{uj\in N}$ are Hermite polynomials and the 6 first polynomials of the Polynomial Chaos are given by:*

$$\begin{aligned}
&\Psi_{(0,0)}(\xi_1,\xi_2) = \psi_0(\xi_1)\psi_0(\xi_2) = 1 \\
&\Psi_{(1,0)}(\xi_1,\xi_2) = \psi_1(\xi_1)\psi_0(\xi_2) = \xi_1 \\
&\Psi_{(0,1)}(\xi_1,\xi_2) = \psi_0(\xi_1)\psi_1(\xi_2) = \xi_2 \\
&\Psi_{(2,0)}(\xi_1,\xi_2) = \psi_2(\xi_1)\psi_0(\xi_2) = \frac{1}{\sqrt{2}}(\xi_1^2 - 1) \\
&\Psi_{(1,1)}(\xi_1,\xi_2) = \psi_1(\xi_1)\psi_1(\xi_2) = \xi_1\xi_2 \\
&\Psi_{(0,2)}(\xi_1,\xi_2) = \psi_0(\xi_1)\psi_2(\xi_2) = \frac{1}{\sqrt{2}}(\xi_2^2 - 1)
\end{aligned} \qquad (7)$$

In the general case, the analytical expression of the coefficients $c_{\boldsymbol{u}}$ of the approximation (2) of $Q(\xi)$ is not available. In [7-11], methods to determine $c_{\boldsymbol{u}}$ are proposed. In our case, we will use the regression method [7]. Once the approximation (4) is available, the global sensitivity analysis of $Q(\xi)$ versus each random variable ξ_j can be performed easily by calculating Sobol coefficients [13]. In the following section, we recall briefly one method proposed in [13] to calculate the Sobol indices from an approximation based on a polynomial chaos expansion.

3. SENSITIVITY ANALYSIS BASED ON SOBOL INDICES

Sobol proposes in [4] to express the quantity of interest $Q(\xi)$ in the following form:

$$Q(\boldsymbol{\xi}) = Q_0 + \sum_{i=1}^{M} Q_i(\xi_i) + \sum_{1\le i_1 < i_2 \le M} Q_{i_1 i_2}(\xi_{i_1}, \xi_{i_2}) + \cdots + Q_{i_1 i_2 \cdots i_M}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_M}) \qquad (8)$$

where Q_0 is a constant and $Q_{i_1 i_2 \cdots i_s}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s}),\ s \le M$, defined such that:

$$\int_{\Theta_{i_k}} Q_{i_1 i_2 \cdots i_s}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s}) \cdot f_{i_k}(\xi_{i_k}) d\xi_{i_k} = 0 \quad \text{with } 1 \le k \le s \quad (9)$$

The decomposition (8) is unique (see [4] and [5]) when $Q(\xi)$ is integrable over $\boldsymbol{\Theta}$ (2). From (9) it can be shown that the functions $Q_{i_1 i_2 \cdots i_s}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s})$ are orthogonal in the sense that:

$$\int_{\boldsymbol{\Theta}} Q_{i_1 i_2 \cdots i_s}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s}) \cdot Q_{j_1 j_2 \cdots j_t}(\xi_{j_1}, \xi_{j_2}, \cdots, \xi_{j_t}) f(\boldsymbol{\xi})\, d\boldsymbol{\xi} = 0 \quad \text{with } (i_1, i_2, \cdots, i_s) \ne (j_1, j_2, \cdots, j_t) \qquad (10)$$

According to the previous properties, it can be easily shown that the variance D of $Q(\xi)$ can be decomposed in the following form:

$$D = \sum_{i=1}^{M} D_i + \sum_{1\le i_1 < i_2 \le M} D_{i_1 i_2} + \cdots + D_{i_1 i_2 \cdots i_M} \qquad (11)$$

where $D_{i_1 i_2 \cdots i_s}$ the partial variances defined by:

$$D_{i_1 i_2 \cdots i_s} = \int_{\boldsymbol{\Theta}} Q^2_{i_1 i_2 \cdots i_s}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s}) f(\boldsymbol{\xi})\, d\boldsymbol{\xi} \qquad (12)$$

The term $D_{i_1 i_2 \cdots i_s}$ is the fraction of the variance of D explained by the interaction between the random variables $(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s})$. Then, the Sobol coefficients are defined by:

$$S_{i_1 i_2 \cdots i_s} = \frac{D_{i_1 i_2 \cdots i_s}}{D} \qquad (13)$$

The Sobol coefficients are positive and their sum is equal to 1. A significant value of a Sobol index $S_{i_1 i_2 \cdots i_s}$ versus the others means that the interaction between the parameters $(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s})$ contributes significantly to the variability of Q(ξ). The number of Sobol indices is equal to 2^M-1 and can be very large if the number M of inputs random is large. In practice, only the M Sobol indices of first order S_i and the M total Sobol indices S_{Ti} are calculated:

$$\begin{aligned} S_i &= \frac{D_i}{D} \\ S_{Ti} &= \sum_{\mathcal{T}_i} S_{i_1 i_2 \cdots i_s} \end{aligned} \qquad (14)$$

where

$$\mathcal{T}_i = \left\{ (i_1, i_2, \cdots, i_s) \mid \exists k,\ 1 \le k \le s,\ i_k = i \right\} \qquad (15)$$

From these both sets of indices, we can conclude that if S_i is significant, the influence of ξ_i is also significant. If S_{Ti} is small, ξ_i has no significant influence. The Sobol indices can be easily estimated using a MSCM by using two distinct samples for the inputs. If an approximation method is used, from the truncated PCE, it is straightforward to approximate the Sobol indices from the coefficients $c_{\boldsymbol{u}}$ (see (16)) [13]. Indeed, the approximation (4) can be rewritten as the form (8) with:

$$Q_{i_1 i_2 \cdots i_s}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s}) = \sum_{\boldsymbol{u} \in \mathcal{T}_{i_1 i_2 \cdots i_s}} c_{\boldsymbol{u}} \Psi_{\boldsymbol{u}}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_M}) \qquad (16)$$

where

$$\mathcal{T}_{i_1 i_2 \cdots i_s} = \left\{ \boldsymbol{u} = (u_1, u_2, \cdots, u_M) \in K^p \mid u_k = 0 \text{ if } k \notin (i_1, i_2, \cdots, i_s) \text{ and } u_k > 0 \text{ if } k \in (i_1, i_2, \cdots, i_s) \right\} \quad (17)$$

From (16) the Sobol coefficients can be deduced directly from the coefficients $c_{\boldsymbol{u}}$.

Example. *If Q is written under the form*

$$Q(\xi_1,\xi_2) = (\sqrt{2}+1)\Psi_{(0,0)}(\xi_1,\xi_2) + \Psi_{(1,0)}(\xi_1,\xi_2) + \Psi_{(0,1)}(\xi_1,\xi_2) + \sqrt{2}\cdot\Psi_{(2,0)}(\xi_1,\xi_2) + \Psi_{(1,1)}(\xi_1,\xi_2) \quad (18)$$

Then, the functions $Q_{i_1 i_2 \cdots i_s}(\xi_{i_1}, \xi_{i_2}, \cdots, \xi_{i_s})$ *are:*

$$\begin{aligned} Q_1(\xi_1) &= \Psi_{(1,0)}(\xi_1,\xi_2) + \sqrt{2}\cdot\Psi_{(2,0)}(\xi_1,\xi_2) \\ Q_2(\xi_2) &= \Psi_{(0,1)}(\xi_1,\xi_2) \\ Q_{12}(\xi_1,\xi_2) &= \Psi_{(1,1)}(\xi_1,\xi_2) \end{aligned} \quad (19)$$

The variance of Q is $D = c_{(1,0)}^2 + c_{(0,1)}^2 + c_{(2,0)}^2 + c_{(1,1)}^2 = 5$. *The Sobol coefficients* $S_{i_1 i_2 \cdots i_s}$ *and the total Sobol coefficients* S_{Ti} *are:*

$$\begin{aligned} S_1 &= \frac{c_{(1,0)}^2 + c_{(2,0)}^2}{D} = \frac{3}{5};\ S_2 = \frac{c_{(0,1)}^2}{D} = \frac{1}{5};\ S_{12} = \frac{c_{(1,2)}^2}{D} = \frac{1}{5} \\ S_{T1} &= \frac{c_{(1,0)}^2 + c_{(2,0)}^2 + c_{(1,1)}^2}{D} = \frac{4}{5};\ S_{T2} = \frac{c_{(0,1)}^2 + c_{(1,1)}^2}{D} = \frac{2}{5} \end{aligned} \quad (20)$$

4. APPLICATION OF THE GLOBAL SENSITIVITY ANALYSIS APPROACH

In the following we will apply the stochastic approach presented above to evaluate the influence of the variability of the behavior law of ferromagnetic materials on the performances of a turboalternator. First, we have to define a stochastic model for the non-linear behavior law. Then, a Stochastic Finite Element problem is solved to determine the flux linkage in function of the excitation current. This characteristic depends on the excitation current I and on the random input, this is consequently a random field . Then, a global sensitivity analysis is carried out to determine which random parameters of the behavior law are the most influential.

4.1. PROBABILISTIC MODEL FOR A RANDOM NON-LINEAR BEHAVIOR LAW

To model the behavior law of the ferromagnetic material, we propose to use a parametrized curve with five parameters (H_1, B_1, H_2, B_2, α) presented in Fig. 1.

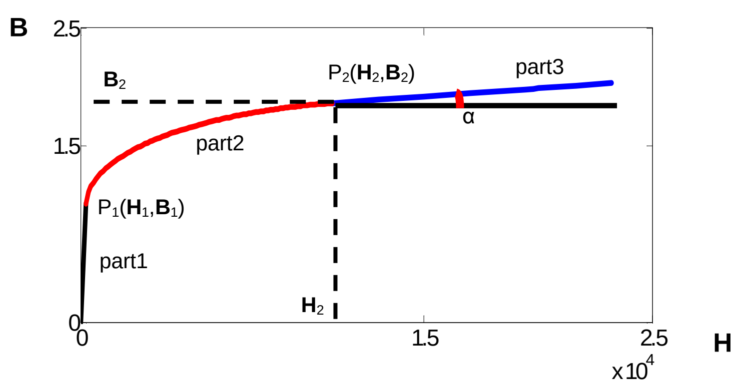

Fig. 1. Parametrized curve B(H)

The curve B(H) is split up in three parts. The part 1 is a straight line determined by the point P_1 with coordinates ($\mathbf{B}_1$,$\mathbf{H}_1$). The part 3 is also a straight line and determined by the point P_2 with coordinates ($\mathbf{B}_2$,$\mathbf{H}_2$) and the angle α. The part 2 is a part of an ellipsoid that verifies the continuity of the curve B(H) and its first order derivative at point P_1 and P_2. Indeed, the part 2 can be represented by the following expression:

$$\frac{(\mathrm{B} - y_0)^2}{b^2} + \frac{(\mathrm{H} - x_0)^2}{h^2} = 1 \qquad (21)$$

where x_0, y_0, b, h are parameters determined in function of B_1, H_1, B_2, H_2, α by imposing the continuity of the curve and its first order derivative at point P_1 and P_2:

$$\begin{aligned} &\frac{(\mathrm{B}_1 - y_0)^2}{b^2} + \frac{(\mathrm{H}_1 - x_0)^2}{h^2} = 1 \; ; \; \frac{(\mathrm{B}_2 - y_0)^2}{b^2} + \frac{(\mathrm{H}_2 - x_0)^2}{h^2} = 1 \\ &\mathrm{B}_1 \frac{2(\mathrm{B}_1 - y_0)}{b^2} + \mathrm{H}_1 \frac{2(\mathrm{H}_1 - x_0)}{h^2} = 0 \; ; \; \sin(\alpha)\frac{2(\mathrm{B}_2 - y_0)}{b^2} + \cos(\alpha)\frac{2(\mathrm{H}_2 - x_0)}{h^2} = 0 \end{aligned} \qquad (22)$$

One can notice that equation (22) can be solved analytically that provides an analytical expression of x_0, y_0, b, h in function of B_1, H_1, B_2, H_2, α (see Appendix I).

When $0 < B_1 < B_2$, $0 < H_1 < H_2$, $0 < \tan(\alpha) < B_1/H_1$, the model presented in the Fig. 1 ensures that the following properties: 1. The curve B(H) is continue and strictly increasing, 2. The first order derivation of the curve B(H) is continue and decreasing.

According to these parameterized model, we propose a stochastic model of a non-linear behavior law based on the parameters (H_1,B_1,B_2,H_2,α) assumed to be independent uniform random variables.

4.2. ELECTRICAL MACHINE

We are interested in the turboalternator geometry of which is given in Fig. 3. The power rate and the nominal voltage of the machine are respectively 1400MW and 20KV. The rotor excitation is fed by a current I. The stator is at no load (no connection to the network). We are interested in the value of the flux Φ flowing through one phase. The behavior of the ferromagnetic material of the stator and the rotor are non-linear. They are represented by two parametric B(H) curves presented in the previous section. The eddy current effects are neglected, so we have to solve a stochastic magnetostatic problem. We are interested in the 3 following cases. In the case 1, the behavior law of the stator material is a random field whereas the one of the rotor is considered deterministic. In the case 2, the B(H) curve of the rotor is assumed to be a random field but not the one of the stator. Finally, in the case 3, we consider that both B(H) curves are random but with a reduced number of random parameters. In order to analyze the sensitivity of the flux versus the random parameters, the flux Φ is approximated by the expansion (4). The coefficients are calculated using a regression method [7] for each value of the current I. While the expression of the flux Φ(I,θ) in the form (4) is available. It is quite easy to characterize the random flux Φ. The proposed scheme is presented in the following flow chart:

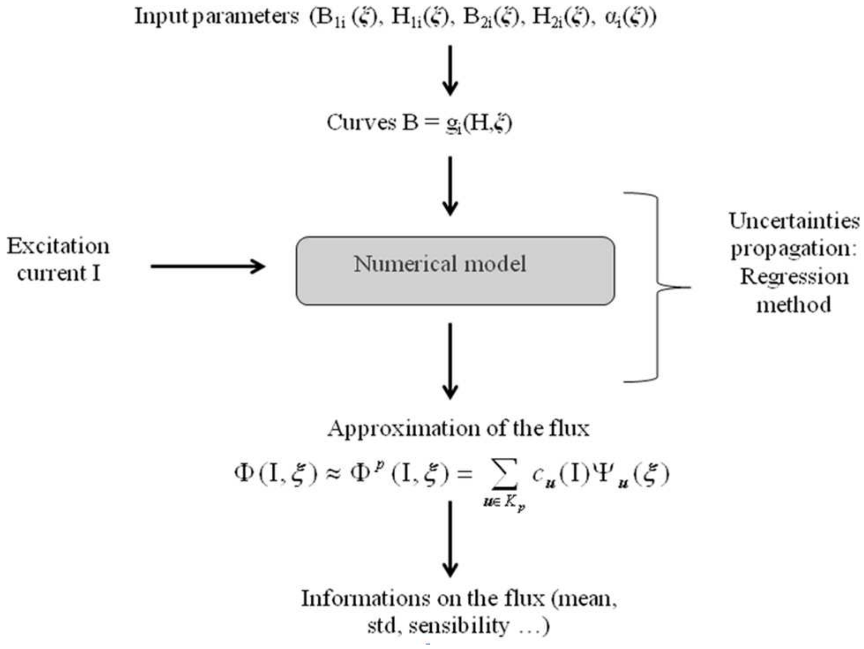

Fig. 2. Uncertainties propagation

B(H) curves of the raw ferromagnetic materials of the stator and the rotor have been measured. Values for the five parameters of the B(H) have been identified from these experimental curves. These values are considered as the mean of the 5 random parameters and are reported in the table I and II for the stator and the rotor respectively. To take into account the uncertainties introduced by the process of characterization, by the origin of the raw material and also by the process of assembling, we have considered these five parameters as uniform random variables with a variability of 15%. The range of variation for each parameter is reported in the table I and II. The B(H) curve corresponding to the identified values for the 5 parameters and also the domain of variability are represented in Fig. 3 and 4 for the stator and the rotor respectively.

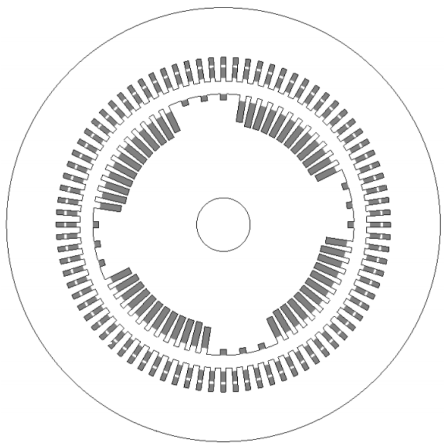

Fig. 3. Geometry of the turboalternator

TABLE I : Information of random variables B_{s1}, H_{s1}, B_{s2}, H_{s2}, α_s

	B_{s1}	H_{s1}	B_{s2}	H_{s2}	α_s
Mean value	1	233	1.94	19440	$2\mu_0$
Lower bound	0.85	198	1.65	16524	μ_0
Upper bound	1.15	268	2.23	22356	$3\mu_0$

TABLE II: Information of random variables B_{r1}, H_{r1}, B_{r2}, H_{r2}, α_r

	B_{r1}	H_{r1}	B_{r2}	H_{r2}	α_r
Mean value	1.11	1639	2.01	13632	$2\mu_0$
Lower bound	0.96	1411	1.73	11739	μ_0
Upper bound	1.26	1866	2.29	15525	$3\mu_0$

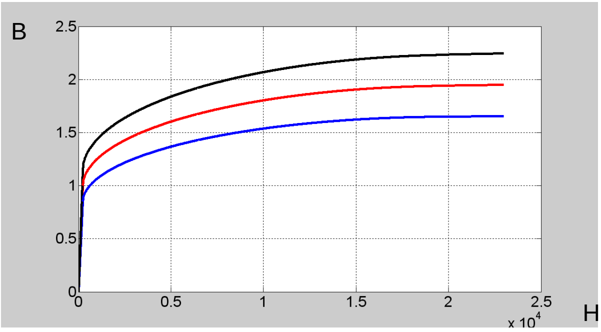

Fig. 4. Curves B(H) of the ferromagnetic material of the stator with the mean (red) and the domain of variability between the blue and black curves

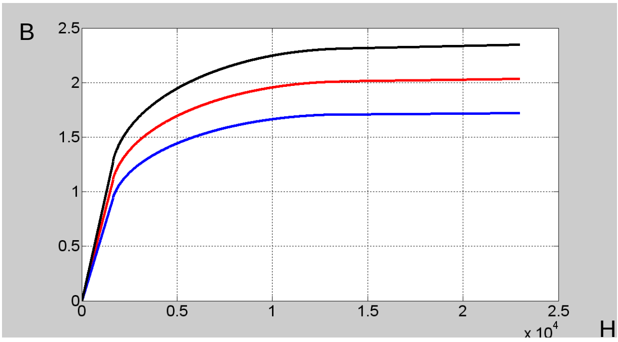

Fig. 5. Curves B(H) of the ferromagnetic material of the rotor with the mean (red) and the domain of variability between the blue and black curves.

A. Case 1

The behavior law of the ferromagnetic material of the rotor is assumed to be deterministic and corresponds to the mean curve presented in Fig.4 . The B(H) curve of the stator is a random field which model has been presented in sections 4.1 and 4.2. First, 200 realizations of the curve $\Phi(I)$ obtained by a Monte Carlo Simulation Method are presented in Fig. 6. We have generated a sample of 200 realizations of the 5-tuple ($B_{s1}(\theta_i)$ $H_{s1}(\theta_i)$, $B_{s2}(\theta_i)$, $H_{s2}(\theta_i)$, $\alpha_s(\theta_i)$), i=1:200. For each realization ($B_{s1}(\theta_i)$ $H_{s1}(\theta_i)$, $B_{s2}(\theta_i)$, $H_{s2}(\theta_i)$, $\alpha_s(\theta_i)$), the flux Φ is calculated for each value of I in order to obtain one curve $\Phi(I,\theta_i)$. The red curves in the correspond to the envelop of the 200 realizations $\Phi(I,\theta_i)$. From Fig. 6, one can observe the magnitude of variability of the flux Φ in function of current I. One can notice that in the linear zone the variability of Φ is very small whereas it is quite large in the saturated area. Fig. 7 represents the mean value and the standard deviation of the flux Φ the excitation current I. One can notice that the standard deviation (image of the flux variability) is an increasing function of the excitation current I confirming the result obtained with the Monte Carlo Simulation (see Fig. 6). From the expansion (2), we also have calculated the evolution of the Sobol indices versus the excitation current. First, it should be noticed that S_i and S_{Ti} are almost equal (see Fig. 8 and Fig. 9) meaning that the contribution to the flux variability of the interactions between parameters is very small. In the following, we will only consider the total Sobol indices that are given in Fig. 8. We can see that for low excitation currents, H_{s1} and B_{s1} are the most influential parameters. Their influence becomes negligible in the saturation area where B_{s2} becomes the most influential, followed by the slope α_s. We can see that the magnetic field H_{s2} has almost no influence even in the saturated area. We consider now the partial variances V_i defined by:

$$V_i = S_{Ti} \cdot D \qquad (23)$$

In Fig. 10, we represent the evolution of these partial variances in function of the excitation current. Thought B_{s1} and H_{s1} are the most influential parameters at low excitation currents, their contribution to the variability of the field remains small. The contribution of the parameter B_{s2} to the variability of the flux is large. Nevertheless, from the value I = 4000 A, one can notice a decreasing influence of B_{s2} and an increasing influence of α_s on the flux Φ. This phenomenon can be explained intuitively by the fact that in the saturated area, the ferromagnetic material state moves progressively from the part 2 of the B(H) curve (see Fig. 1) to the part 3 where the influence of α_s increases.
The almost negligible influence of B_{s1}, H_{s1} and H_{s2} can be shown again by plotting the mean value and the standard deviation of the flux Φ in function of the current I in the case where B_{s2} and α_s are fixed equal to their mean value (see Fig. 11). One can notice that the variation of Φ is much smaller than in the Fig. 7.

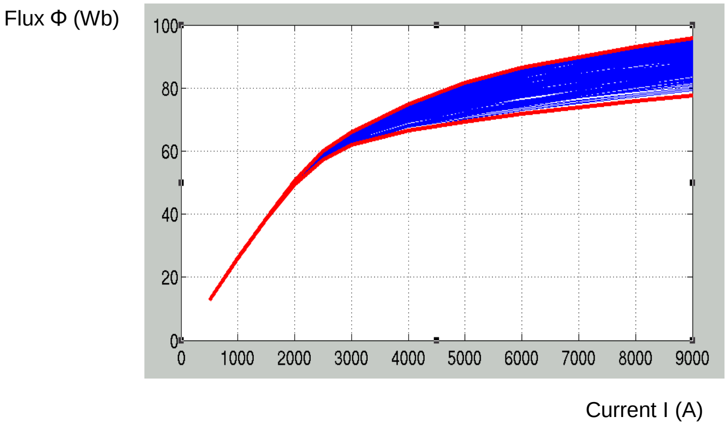

Fig. 6. 200 realizations of the flux Φ versus the excitation current I

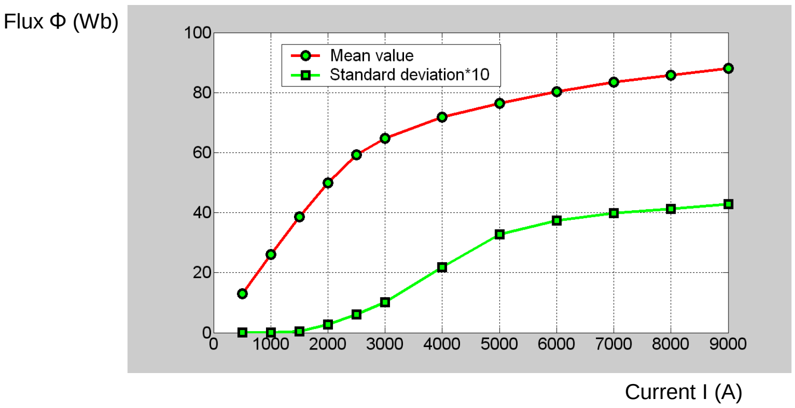

Fig. 7. Mean value and standard deviation of the flux Φ versus the excitation current I

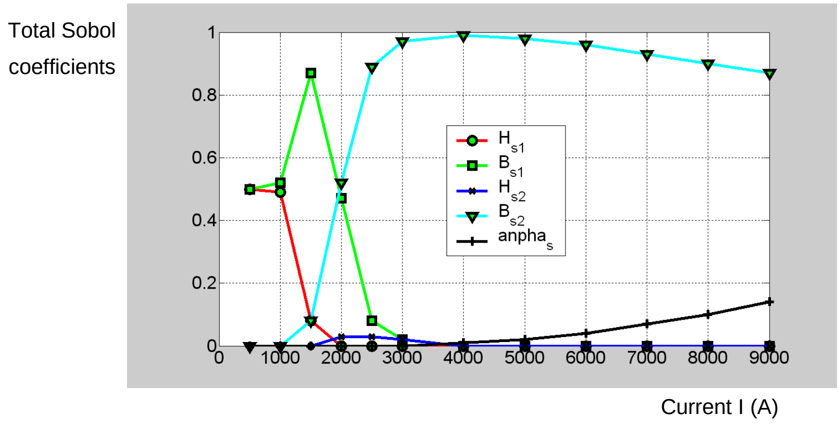

Fig. 8. Total Sobol coefficients of each random input data B_{s1}, H_{s1}, B_{s2}, H_{s2}, α_s

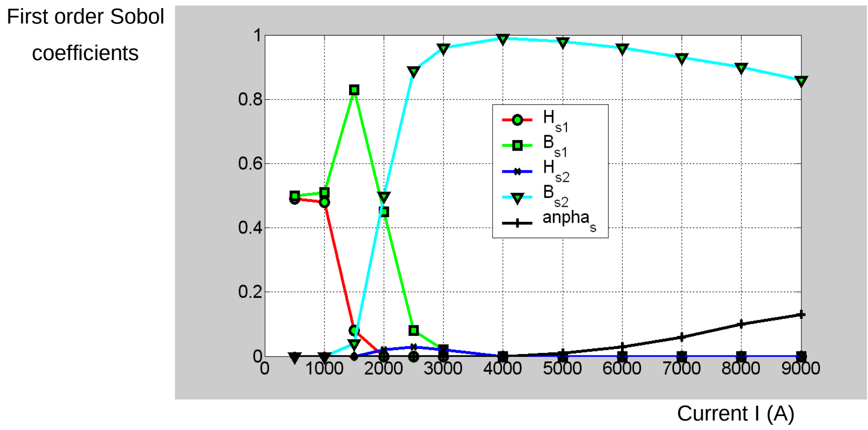

Fig. 9. First order Sobol coefficients of each random input data B_{s1}, H_{s1}, B_{s2}, H_{s2}, α_s

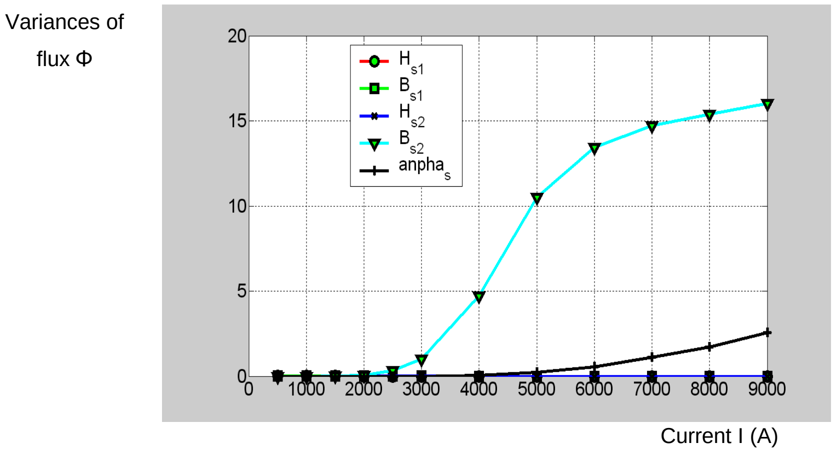

Fig. 10. Partial variance contributed by each random input data B_{s1}, H_{s1}, B_{s2}, H_{s2}, α_s

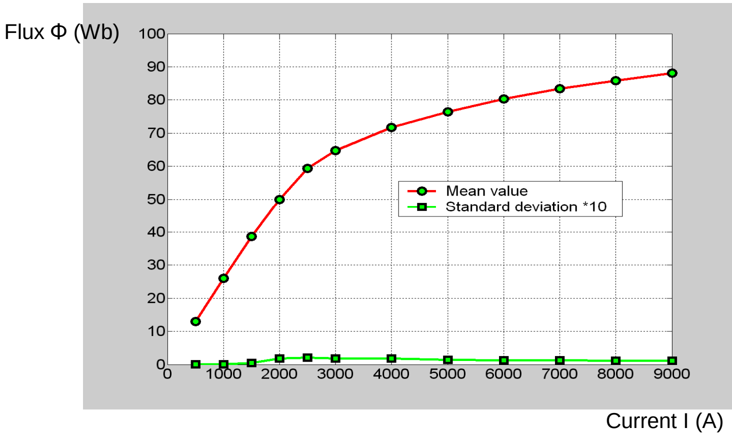

Fig. 11. Mean value and standard deviation of the flux Φ versus the excitation current I (B_{s2}, α_s are fixed)

B. Case 2

In this case, the B(H) curve of the stator is assumed to be deterministic and equal to the mean curve represented in Fig.3. The B(H) curve of the rotor is a random field which model has been presented in sections 4.1 and 4.2. In Fig. 12, the evolutions of the mean and the standard deviation of the flux in function of the current are given. This figure can be compared to the Fig. 7 corresponding to the case 1. We can see that the evolution of the mean of the flux is almost the same. The difference appears on the standard deviation. The variability of the flux is higher in the case 2 for excitation current I lower than 4000A. Then, the variability of the flux becomes higher in the case 1. We could expect that the variability of the B(H) curve of the rotor will contribute the most for the values of the excitation current lower than 4000A.

Concerning the sensitivity analysis, the results obtained in the case 2 are very similar to the case 1. The flux Φ is less sensitive in the linear zone than in the saturated area and the influence of B_{r1}, H_{r1}, H_{r2} on the variability of the flux Φ is very small compared to the one of B_{r2}, α_r.

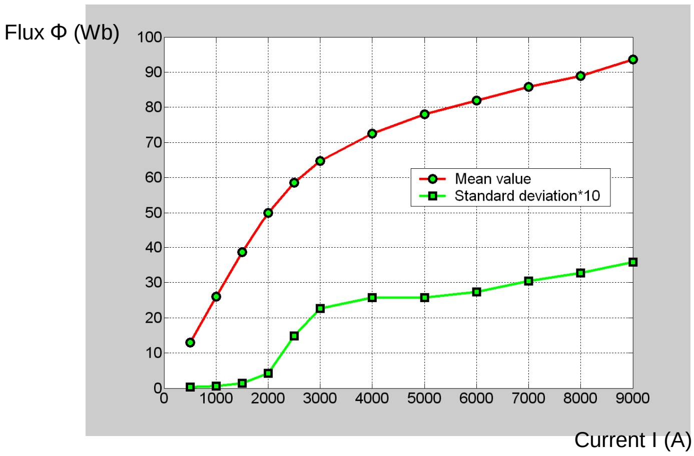

Fig. 12. Mean value and standard deviation of the flux Φ versus the excitation current I

C. Case 3

The most influential random variable are, in cases 1 and 2, the ordinate B_2 of the point 2 and the slope α (see Fig. 1). In the case 3, we consider that both materials in the stator and in the rotor are random but the randomness of the curves B(H) is borne only by the random variables B_{r2}, α_r and B_{s2}, α_s. Other input parameters B_{r1}, H_{r1}, H_{r2}, B_{s1}, H_{s1}, H_{s2} are assumed to be deterministic. This assumption is not mathematically rigorous but in practice, it seems to be reasonable. It allows reducing the number of input parameters to 4 instead of 10 and so, it allows to dramatically reduce the computation time. The input random parameters B_{r2}, α_r and B_{s2}, α_s are assumed to be uniform and are the same as in the previous cases (see TABLES I and II).

The steps of calculation are the same as in the case 1. First, we have simulated a sample of 200 realizations of the curve $\Phi(I,\theta)$. The dispersion is a little higher than in the case 1 (see Fig. 13 and Fig. 6) where the variability is only borne by the B(H) curve of the stator. Afterwards, the flux Φ is approximated by the expansion (4) and then, the mean value, standard deviation of Φ and the Sobol coefficients can be deduced.

Fig. 14 presents the evolution of the mean and the standard deviation in function of the excitation current. We can see that, compared to the case 1, the variability has increased for excitation current values between 2000A and 5000A. It means that the contribution to the variability of $\Phi(I,\theta)$ is mainly due to the variability of the B(H) curve of the rotor for high excitation current in the range [2000A, 5000A].

In this case, the difference between the first order and the total Sobol coefficients is also very small. Therefore, we consider only the Total Sobol coefficients (Fig. 16). We have also drawn the evolution of the partial variances (see (21)) in function of the current I in Fig. 15. It confirms the fact noticed above where the parameter B_{r2} is the most influential up to I=4200A. Above this value of I, the parameter B_{s2} becomes the most influential. From I=6000A, one can notice the increasing influence of α_r and α_s with a decreasing influence of B_{r2} and B_{s2}, meaning that the saturated zone of the rotor and the stator moves from the part 2 toward the part 3 of the B(H) curve (see Fig. 1).

It appears, using this approach, that in order to reduce the variability of the output $\Phi(I,\theta)$ in the range of the study [0A, 9000A], one should reduce the variability on the parameters B_{r2} and B_{s2}, meaning that the measurement of the quantity B should be carefully done in the range [1.6T, 2.2T] for B_{s2} and [1.7T, 2.3T] for B_{r2}. In the area [0A, 9000A], we can see that the parameters H_{s2} and H_{r2} have almost no influence on the variability of the flux. We can see also that the B(H) curve has few effects for I lower than 2000A. This statement can be useful to specify a set up in situ to characterize directly the B(H) in the machine like proposed [21] and [22]. The measurements of B should be accurate whereas a loss of precision on the measurement of H can be accepted (due to parasitic air gap for example).

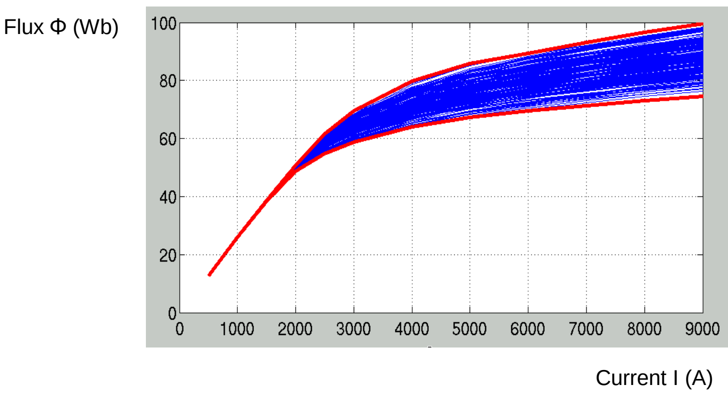

Fig. 13. 200 realizations of the flux Φ in function of the excitation current I

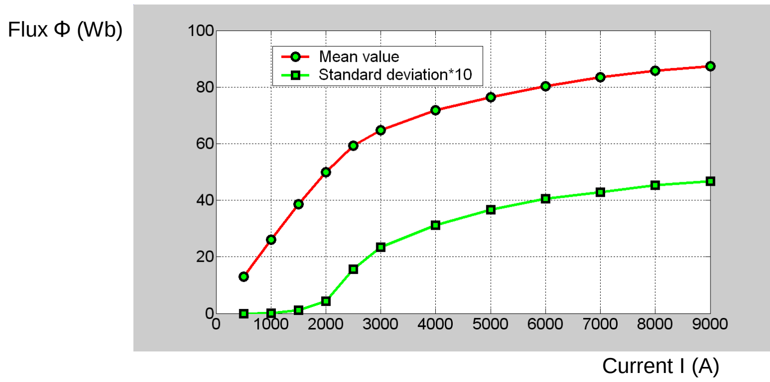

Fig. 14. Mean value and the standard deviation of the flux Φ in function of the excitation current I

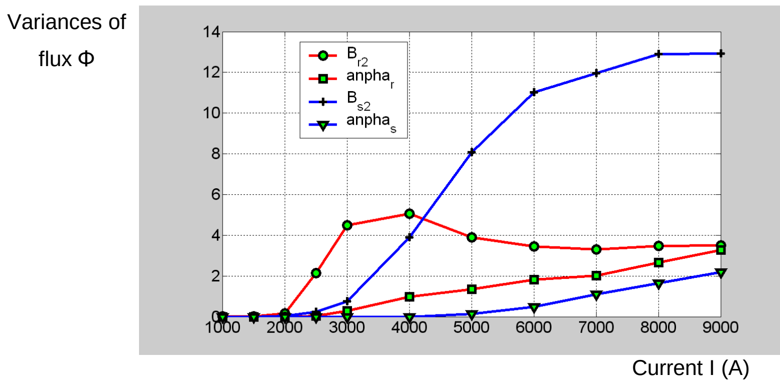

Fig. 15. Partial variances contributed by each random input data B_{s2}, α_s, B_{r2}, α_r

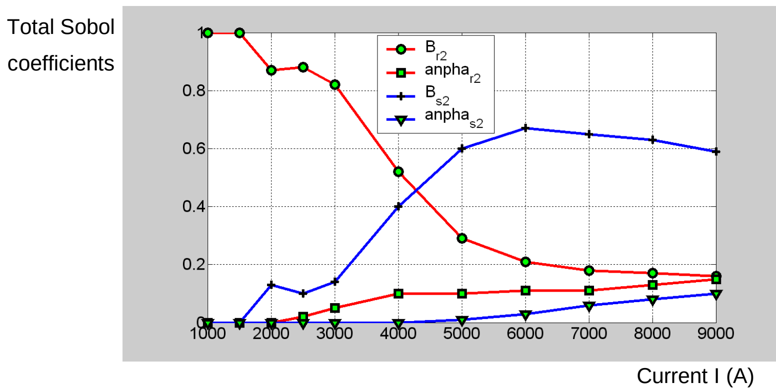

Fig. 16. Total Sobol coefficients correspond to each random input data B_{s2}, α_s, B_{r2}, α_r

It should be mentioned that the results concerning the influence of the parameter could be qualitatively predicted just considering physical considerations. However, the additional value of the stochastic approach relies on the quantitative evaluation of the influence. We can see clearly that the point P1 in its range of variability has no influence on the flux linkage. We could have expected a fewer influence than the point P2 and the slope α but not no effect. The fact that the parameters B_2 (particularly the one of the stator) are more influential in the range of study than α the slope was not predictable also. This representation of the B(H) curve coupling with the stochastic approach enables to determine which range of the B(H) curve should be modeled as accurately as possible to limit the uncertainty on the output. The stochastic approach can be also used with other representation of the B(H) curve (Langevin for example) and to determine which parameters of the model should be carefully identified.

5. CONCLUSION

In this paper, the influence of the non-linear material behavior law on the performance of a turbogenerator have been analyzed. A stochastic model of the non-linear curve B(H) has been proposed. Using this model, the randomness of the curve B(H) can be borne only by a finite number of random variables. From the proposed model, a global sensitivity analysis based on Sobol coefficients have been performed. The obtained results show that the influence of the input parameters on the performance is not the same for all the parameters and depend on the level of saturation of the machine. At low excitation current, the variability on the B(H) curve has almost no effect on the flux linkage. The variability of the flux linkage is maximum when the machine is saturated (at high excitation current). The stochastic approach enables to characterize quantitatively this variability by determining the standard deviation. The global sensitivity analysis based on the Sobol approach allows to determine the most influential parameters of the B(H) curve. It appears that the magnetic flux density B value is the most influential but not the magnetic field H in the saturation area. The proposed approach provides the area where the input parameters are the most influential and then allows to act in order to reduce their variability by increasing the accuracy of the measurement in the corresponding area.

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Appendix I

In this appendix, the expressions of the solutions of (22) are given. If we denote:

$$
\begin{aligned}
&\mu_1 = \frac{\mathrm{B}_1}{\mathrm{H}_1};\ in\_\mu_1 = \frac{1}{\mu_1};\ \ \mu_2 = \tan(\alpha);\ \ in\_\mu_2 = \frac{1}{\mu_2}\\
&c_1 = \frac{\mathrm{H}_1 - \mathrm{H}_2}{\mu_1 - \mu_2};\ c_2 = \frac{\mu_2(\mathrm{B}_1 - \mathrm{B}_2)}{\mu_1 - \mu_2};\ c_3 = \frac{(\mathrm{B}_1 - \mathrm{B}_2)}{in\_\mu_1 - in\_\mu_2};\\
&c_4 = \frac{in\_\mu_2(\mathrm{H}_1 - \mathrm{H}_2)}{in\_\mu_1 - in\_\mu_2};\ c_5 = \frac{\mu_1(\mathrm{B}_1 - \mathrm{B}_2)}{\mu_1 - \mu_2};c_6 = \frac{in\_\mu_1(\mathrm{H}_1 - \mathrm{H}_2)}{in\_\mu_1 - in\_\mu_2}\\
&k = \frac{c_2^2 - c_5^2 + 2c_3(c_4 - c_6)}{c_6^2 - c_4^2 + 2c_1(c_5 - c_2)}
\end{aligned}
\tag{24}
$$

The solutions of (22) are given by:

$$
\begin{aligned}
&h = c_1^2 k + 2c_1c_2 + \frac{c_2^2}{k} + (\frac{c_3}{k} + c_4)^2\\
&b = k \cdot h\\
&y_0 = k \cdot c_1 + \mathrm{B}_1 + c_2\\
&x_0 = \frac{c_3}{k} + c_4 + \mathrm{H}_1
\end{aligned}
\tag{25}
$$